2.2 Binomial Theorem

Expand:-

$(a + b)^0 =$

$(a + b)^1 =$

$(a + b)^2 =$

$(a + b)^3 =$

$(a + b)^4 =$

D Sequences and series

D1

Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and $n\mathrm{C}r$; link to binomial probabilities.

Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left|\frac{bx}{a}\right| < 1$ (Proof not required.)

Teaching guidance

Students should be able to:

- answer questions requiring the full expansion of expressions of the form $(a+bx)^n$, where n is a small positive integer
- find the coefficients of particular powers of x (complete expansion not required)
- understand factorial notation.

D Sequences and series

D1

Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and nCr; link to binomial probabilities.

Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left|\frac{bx}{a}\right| < 1$ (Proof not required.)

Notes

- The notations $\binom{n}{r}$, ${}_nC_r$ and ${}^nC_r$ must all be recognised. Any of these may be used.
- The x in $(a+bx)^n$ may be a simple function of x, eg $\left(2-\frac{1}{x}\right)^4$

omial Expression – An expression with two ter

$(a + b)^0 = 1$ $\qquad 1$

$(a + b)^1 = a + b$ $\qquad 1a + 1b$

$(a + b)^2 = (a + b)\,(a + b)$ $\qquad 1a^2 + 2ab + 1b^2$

$(a + b)^3 = (a + b)^2\,(a + b)$ $\qquad 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a + b)^4 = (a + b)^3\,(a + b)$ $\qquad$ **$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b$**

$(a + b)^5 =$

$(a + b)^6 =$

omial Expression – An expression with two ter

$(a + b)^0 =$ $\mathbf{1}$

$(a + b)^1 =$ $\mathbf{1}a + \mathbf{1}b$

$(a + b)^2 =$ $\mathbf{1}a^2 + \mathbf{2}ab + \mathbf{1}b^2$

$(a + b)^3 =$ $\mathbf{1}a^3 + \mathbf{3}a^2b + \mathbf{3}ab^2 + \mathbf{1}b^3$

$(a + b)^4 =$ $\mathbf{1}a^4 + \mathbf{4}a^3b + \mathbf{6}a^2b^2 + \mathbf{4}ab^3 + \mathbf{1}b^4$

$(a + b)^5 =$

$(a + b)^6 =$

omial Expression – An expression with two ter

$(a + b)^0 =$ 1

$(a + b)^1 =$ $1a + 1b$

$(a + b)^2 =$ $1a^2 + 2ab + 1b^2$

$(a + b)^3 =$ $1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a + b)^4 =$ $1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$(a + b)^5 =$

$(a + b)^6 =$

omial Expression – An expression with two ter

$(a + b)^0 =$ 1

$(a + b)^1 =$ $1a + 1\mathbf{b}$

$(a + b)^2 =$ $1a^2 + 2a\mathbf{b} + 1\mathbf{b}^2$

$(a + b)^3 =$ $1a^3 + 3a^2\mathbf{b} + 3a\mathbf{b}^2 + 1\mathbf{b}^3$

$(a + b)^4 =$ $1a^4 + 4a^3\mathbf{b} + 6a^2\mathbf{b}^2 + 4a\mathbf{b}^3 + 1\mathbf{b}^4$

$(a + b)^5 =$

$(a + b)^6 =$

omial Expression – An expression with two ter

$(a + b)^0 =$ 1

$(a + b)^1 =$ $1a + 1b$

$(a + b)^2 =$ $1a^2 + 2ab + 1b^2$

$(a + b)^3 =$ $1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a + b)^4 =$ $1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$(a + b)^5 =$ $1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b$

$(a + b)^6 =$ $1a^6 + 6a^5b + 15a^4b^2 + 20^3b^3 + 15a^2b^4 + 6ab^5 + 1b$

^nC_r Notation:

The numbers in Pascal's Triangle can be generated using the formula ^nC_r

n = 5: 1 5 10 10 5 1

(6th row) $^5C_0$ $^5C_1$ $^5C_2$ $^5C_3$ $^5C_4$ $^5C_5$

ou can calculate any particular value using the following formula

In general: $^nC_r = \dfrac{n!}{r!\,(n-r)!}$ sometimes ^nC_r is written as $\binom{n}{r}$

Example: $^5C_2 = \dfrac{5!}{2!\,(5-2)!} = \dfrac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}\,(3 \times 2 \times 1)} = \dfrac{60}{6} = 10$

n = 6: $^6C_0$ $^6C_1$ $^6C_2$ $^6C_3$ $^6C_4$ $^6C_5$ $^6C_6$

(7th row)

1 6 15 20 15 6 1

2.2 The Binomial Theorem

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

2.2 The Binomial Theorem

Example 1

Find the expansion of

$$¿\binom{3}{0}(1)^3(x)^0$$

$$+\binom{3}{1}(1)^2(x)^1$$

$$+\binom{3}{2}(1)^1(x)^2$$

$$+\binom{3}{3}(1)^0(x)^3$$

$$¿\mathbf{1+3}\square\mathbf{+3}\square^{\mathbf{2}}\mathbf{+}\square^{\mathbf{3}}$$

2.2 The Binomial Theorem

Example 2

Find the expansion of

$$¿\binom{5}{0}(3)^5(-2x)^0$$

$$+\binom{5}{1}(3)^4(-2x)^1$$

$$+\binom{5}{2}(3)^3(-2x)^2$$

$$+\binom{5}{3}(3)^2(-2x)^3$$

$$+\binom{5}{4}(3)^1(-2x)^4$$

$$+\binom{5}{5}(3)^0(-2x)^5$$

$$¿243-810▯+1080▯^2-720▯^3+240▯^4-32▯$$

2.2 The Binomial Theorem

Example 3

Find the first four terms in the binomial expansion of in ascending powers of .

$$¿\binom{10}{0}(1)^{10}(2x)^{0}$$

$$+\binom{10}{1}(1)^{9}(2x)^{1}$$

$$+\binom{10}{2}(1)^{18}(2x)^{2}$$

$$+\binom{10}{3}(1)^{7}(2x)^{3} + \dots$$

$$¿\,\mathbf{1+20□+180□^{2}+960□^{3}+\dots}$$

2.2 The Binomial Theorem

Example 4

Find the first four terms in the binomial expansion of in ascending powers of .

$$¿\binom{6}{0}(10)^6\left(\frac{1}{2}x\right)^0$$

$$+\binom{6}{1}(10)^5\left(\frac{1}{2}x\right)^1$$

$$+\binom{6}{2}(10)^4\left(\frac{1}{2}x\right)^2$$

$$+\binom{6}{3}(10)^3\left(\frac{1}{2}x\right)^3 + \ldots$$

$$¿1000000 - 300000\,□ + 37500\,□^2 - 2500\,□^3 + \ldots$$